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MODELING TRUCK PARKING ALONG THE HIGHWAY (PHASE II)

Final Report

by

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DISCLAIMER

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EXCUTIVE SUMMARY

Regulated by the Hours-of-Service law, truckers must find places to rest during their duty (mainly driving) hours periodically for the sake of safety. Without an adequate parking spot, truckers are often found either to park at illegal locations or violating the driving hours. Truck parking is an issue of national concern. The parking capacity provision is due to both the public and private sectors. A key point is to understand the truck parking demand and come up with an adequate parking capacity supply to ensure availability of parking space when needed by considering traffic peaking, network topology and driver's preferred driving hours when it's within the legal range.

The objective of this study is to continue on the success of Phase I of this effort by studying the relation between truck volume and parking space density in a simulation environment as phase I. The truck space availability issue is essentially one between volume and density subject to boundary conditions. In phase I, the team developed a computer simulation program to numerically show the relationship between volume, driving behavior, and parking capacity need, which illustrates the inherent relationship between behavior, truck volume, and parking capacity needs. Based on the vast literature search in Phase I, the study borrows some ideas outside the transportation field to analytically derive the truck parking density model which inherent reveals the relationship between truck volume, driving behavior and truck parking capacity need.

This study models the parking demand density long the highway and discuss location decision of the parking areas. Several closed form formulas of estimating truck parking density are developed based on the renewal process. The formulas are further improved by considering road end effect, network effect and different probability distribution of driving and resting. We use data from the Florida Department of Transportation to illustrate the use and implications of our proposed models. We hope the results may facilitate the understanding and solution of the truck parking issues for this major mode of freight transportation.

1.0 INTRODUCTION

Trucking transportation carries the most amount of goods in terms of value. It employs a large number of trucks running on the nation's highway system compared to other modes such as rail and waterway. Truckers often need to drive long hours for the shipping services, subject to fatigue and fatal crashes. For the sake of safe and efficient operations, U.S. truck drivers are regulated by the Hours-of-Services law, which limits truckers' consecutive driving time up to 8 straight hours and not to drive exceeds 11 hours within any consecutive 24 hours of tthe time. Thus, truck drivers must find a rest area to comply with the regulation during their trips, especially for those drivers on inter-city travels. The hours-of-service law ensures that drivers be in good condition while driving. However, Due to the unavailability of truck parking space at locations, it is needed, truckers are often found parking illegally on highway ramps or other unsafe spots. Or some truckers are caught driving beyond the hours limit, which significantly contributes to the highway's fatal rate. As a result, sufficient truck parking capacity provision along the highways is essential. Almost every state has conducted truck parking studies in an effort mainly to address the truck parking space shortage problem. The difficulty resides in the fact that truck volumes, although varies across regions, fluctuate with time of year and time of day driven by economic activities even in one region. The intuitive observation is that a higher volume demands more parking space statistically. The truck parking issues have caught wide attention as a national concern for many years. States and Federal's truck parking study has been surveyed in the Phase I report. Most of the studies conduct surveys to the truck drivers. The results help the decision-makers to know the situation better and start solving the problem. However, most of the study focuses on their unique local characteristics. They have not put forward a general or theoretical solution methodology to identify the place of suffering parking shortage or estimate the number of parking spots needed.

In phase one, we tried to build a pure simulation method to study the effect of hours-of-service regulation on truck drivers' behaviors. The model simulates truck drivers' behavior and adjusts different parameters, such as the interval between parking facilities, various distribution of truck drivers' starting time, and truck speed. We discover the effects and extent of the effect of the above-mentioned parameter to the number and location of the parking facilities. The objective of this study is to continue the success of Phase I of this effort by continuing to study the relation between truck volume and parking space density in a simulation environment as phase I.

The truck parking space availability issue is essentially one of balance between truck volume and parking density subject to boundary conditions. In phase two, we try to model the truck drivers' behaviors in a theoretical way. In this phase, we model the problem from truck drivers' perspective and aim to associate the location and capability of parking facilities with the remaining driving time. In a statistical way of modeling, it is easy to capture the general effect of Hour-of-Service regulation upon the interval and capacity of truck parking facilities. The target of the second phase is to unveil the parking space density and truck volume relationship. To be general and capture the essentials of the relationship, we give up considering the detailed factors of the highway segments and use a statistical way to model it.

In addition, Phase I has also led to a literature review that surprisingly identifies cases in telecommunication areas that model the relationship between cell phone travelers' volume and ground station service capacity and spacing along a highway. However, that study is inherently different from this truck parking problem, although it sheds light to the problem. Our goal in Phase II is to build on the literature reviewed and also utilize the simulation tool developed in Phase I to analytically derive the inherent analytical relationship between truck volume, driving behavior and truck parking capacity need in a hope that policy makers may use to examine adequacy of truck parking space within their jurisdiction areas.

2.0 LITERATURE REVIEW

We have summarized much literature that use survey method to identify and improve truck parking shortage in the Phase I report. As indicated before, we will further focus on the theoretical modeling literature in this report. There is generally a scant literature that develop general analytical methodologies for this problem over the past decades. We are providing a brief review of the major results relevant to our study here.

Simulation is a common method to discover the trucker's behavior and the interaction with the parking spot. *Steenberghen et al.* use multinomial Logit model to describe parking behaviors and develop an agent-based model (ABM) used to simulate the local parking and traffic situation under different parking-management conditions (Steenberghen et al., 2012).

Munuzuri et al. develop a microscopic traffic simulator parking planning related to freight transport and private traffic in the urban area (Munuzuri et al., 2002). *Nourinejad et al.* proposes a econometric parking choice model to evaluate the potential impact of truck parking in the urban area. The model shows that reserved freight parking may reduce mean searching time for commercial vehicles in urban area, but it may result in higher search time and walking time for passenger car drivers (Nourinejad et al., 2014).

Srivastava et al. develop an online GIS survey tools for collecting the location information of areas with truck parking capacity shortages. They adopt location clustering algorithm to the data and find that outskirts of major urban areas have a higher probability of suffering from truck parking shortages, reflecting the need of staging for next day delivery (Srivastava et al., 2012).

There is limited literature focusing on developing an analytical method of truck parking problems. In 1996, FMCSA evaluated the adequacy of rest parking facilities and regulation in 48 states along interstate highways by observing driver’s actual rest time and driving hours by interviewing industry workers (Trucking Researcch Institute, 1996). As the pioneer research on the truck parking problem, the study identified primary demand-related factors and supply-related factors based on the parking usage of the public rest areas. A linear capacity utilization model was developed and calibrated to assess the utilization and potential needs for truck parking at individual rest areas.

As the successor of the previous report, *Pécheux et al.* developed analytical models to estimate the demand for truck parking spaces, which is widely used in later master plan and studies (Pécheux, et al., 2002). They first assessed the status of nationwide public rest area and then calibrates the truck parking demand model for a designated highway segment while ignoring the association with a single parking facility’s characteristic such as capacity and amenities. The model considers effects brought by seasonal, short-haul to long-haul trucking ratio and time spent at a shipper/receiver, which does shed light to later analytical studies of this subject.

Similar to the United States, the European Union also experiences issues due to the truck parking shortage problem. *Heinitz et al.* (Heinitz et al., 2009) developed a demand modeling approach for limited truck parking facilities from the perspective of drivers. Compared to prior studies, this study made a stride forward and developed models using varying traffic flow instead of average daily traffic. Other papers also proposed various mathematic models , including approximate methodology (Jaller et al., 2013), econometric choice model (North Jersey Transportation and North Jersey Transportation Planning Authority, 2009) and demand modeling (Garber et al., 2002, Tam et al., 2000).

There are a few distinct studies that we feel a need to highlight here individually. *Koo et al.* adopted a case specific reasoning approach, and developed a decision support system for determining the optimal sizes of 173 new expressway service areas (ESAs) with a goal of their profitability (Koo et al., 2014). *Richard Arnott* (Arnott et al., 1999) first presented a stochastic model with focusing on drivers' search for a vacant parking space in a spatially homogeneous metropolis. They examine stochastic stationary-state equilibrium and optima in the model.

Ideas or literature from other fields may shed light on our study problem here, although the problems in other fields may appear distinctly different. For example, Massey *et al.* in the telecommunication area modeled the relationship between cell phone travelers volume and ground station service capacity and spacing, also along a highway (Massey et al., 2009). *Tavafoghi et al.* propose a parking capacity prediction method that provide a real-time probabilistic forecast of parking occupancy for an arbitrary forecast horizon based on the queuing model, time series model and LSTM model (Tavafoghi et al., 2019). We are inspired to build a stochastic model to estimate the potential parking needs on any point of a highway. The next section introduces the details of the model.

3.0 METHODOLOGY

The truck parking study problem may be re-capped here as follows. Trucks travel on the highway from origins to their destinations. Truck drivers are subject to the Hours-of-Service regulation as specified at the beginning of the paper and need to stop at rest areas for rest in order to comply with the regulation. Rest areas are provided along the highway to truck drivers. Trucks may park at an available spot when the drivers feel a need to. The goal of this study is to develop a model for the estimation of truck parking capacity needs along the highway. To facilitate modeling, we assume trucks enter a freeway after they have traveled a known period of time, the length of which follows a probability distribution. The length of driving before a break assumes a given probability distribution as well. The rest time a driver takes at a rest area is random and also follows a probability distribution.

3.1 ASSUMPTIONS AND PRELIMINARIES

The focus of our study here is to model truck parking needs along roadways as related to a set of highway and traffic parameters. For the ease of modeling, we start with a disaggregate approach that focuses on an individual trucker's operation.

In this model, the driver only experiences two types of periods alternately: drive and rest. We assume the driving time H and rest time R are random and each follows a particular distribution, reflecting variations among truckers regarding driving and rest. We model this problem by assuming truck traffic happens on a line of highway with infinite length to reach a steady state of parking needs. Along this highway, the driver's driving time in all the driving periods after rests follows *i.i.d* distribution. So, rest times also follow *i.i.d.* The ultimate result of the result is expected to be aggregate. This report builds on the basic concept of the renewal process.

3.2 RENEWAL PROCESS

A renewal process is a point process in which the inter-event intervals are independent and drawn from the same probability density (Resnick et al., 2002). More specifically, it is defined in theorem 1.

Theorem 1 *Let $\{Y_n\}$ be independent non-negative finite random variables with Y_n has a Cumulative distribution function (CDF) F, $n \geq 1$, and $S_n = Y_1 + Y_2 + \ldots Y_n$. Where S_n is called a renewal sequence. Each S_n is a renewal epoch or renewal time. y_n is called an inter-arrival time or waiting time. The renewal sequence is pure if $Y_0 = 0$ with probability 1.*

The study of renewal processes can be described by a special type of integral equation known as a renewal equation. Renewal equations almost always arise by conditioning on the time of the first arrival and by using the defining property of a renewal process—the fact that the process restarts at each arrival time, independently of the past. The definition of renewal equation is in theorem 2.

Theorem 2 *let* $N(t) = \sum_{n=0}^{\infty} \mathbf{1}_{0 \le S_n \le t} = max\{n : S_{n-1} \le t\}$ *be a renewal process. It denotes the number of renewal epochs in [0, t]. Let V(t) = E(N(t)) be the renewal function.*

It is noteworthy that $N(t) > n$ equivalents to $S_n \le t$, so $S_{N(t)-1} \le t \le S_{N(t)}$. Also $V(t) = E\big(N(t)\big) = \sum_{n=0}^{\infty} P(N(t) > n) = \sum_{n=0}^{\infty} P(S_n \le t) = E\big(\sum_{n=0}^{\infty} 1_{S_n \le t}\big)$ is non-decreasing. For proper distribution CDF $F(F(t) \to 1$ as $t \to \infty)$, then $V(t) \to \infty$ as $t \to \infty$. From theorem 1 and theorem 2, we can see that the distribution of the sum of known random variables is essential. Next, we introduce the definition of convolution.

Theorem 3 *Suppose function U is non-negative, non-decreasing and right continuous and function g is bounded on finite intervals. Define the convolution*

$$u * g(t) = \int_0^t g(t-x)U(dx) \tag{1}$$

for appropriate function g (usually at least g is bounded on finite intervals).

If function *f and g* are density functions for probability distribution *F and G*, respectively,

$$F * g(t) = \int_0^t g(t-x)f(x)dx\,, \quad t \ge 0 \tag{2}$$

Equation (2) denotes the density of $X + Y$ if both X and Y are non-negative random variables. Then we generalize the idea and introduce the sum distribution of n independent random variables.

Theorem 4 *Suppose* X_i, F_i, $i \ge 1$*are independent non-negative random variables. Then* $S_n = y_1 + y_2 + \dots y_n \sim F = F_1 * F_2 * \dots * F_n$. *If* Y_i *are independent identically distributed with distribution* F_i, *then it can be written as* $S_n \sim F^{*n}$

We can easily write the renewal function for the sum variables: $V(t) = E\big(N(t)\big) = \sum_{n=0}^{\infty} F_0 * F_t^{*n}, U(t) = \sum_{n=0}^{\infty} F_t^{*n}$, so in general case, $V(t) = F_0 * U(t)$. Then, we can introduce elementary renewal theorem:

Theorem 5 *(Elementary Renewal Theorem) Set* $\mu = E(Y_1)$, y_i *is i.i.d,* $i \ge 1$ *and assume* $P(y_0 < \infty) = 1$. *Assume* $X_0 = 0$, *Then*

$$\underset{t\to\infty}{Lim}\frac{V(t)}{t} = \underset{t\to\infty}{lim}\frac{U(t)}{t} = \frac{1}{\mu} \quad (3)$$

Proof: first we compute

$$P(N(t) \to \infty) = \lim_{n\to\infty} P\,(N(t) > \text{n for some t}) = \lim_{n\to\infty} P\,(S_n < \infty)$$

$$= \lim_{n\to\infty} P\,(y_0 < \infty, y_1 < \infty, \dots, y_n < \infty)$$

$$= \lim_{n\to\infty} \big(P(y_1 < \infty)\big)^n$$

The Figure 1 display the whole process:

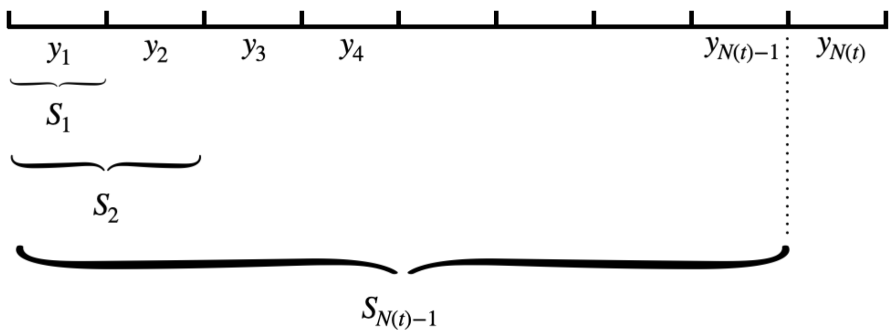

Figure 1: Sample plot for SN(t) and N(t)

$\frac{S_{N(t)}}{N(t)}$ denotes the average interval length, thus if $P(y_1 < \infty) < 1$, then $P(N(t) \to \infty) = 0$ and $\frac{N(t)}{t} \to \frac{1}{\mu}\left(\frac{1}{\infty}\right)$ But if $P(y_1 < \infty) = 1$, then $P(N(t) \to \infty) = 1$ and by the Strong Law of large numbers, we have

$$\frac{S_{N(t)}}{N(t)} \to \mu \quad \frac{S_{N(t)-1}}{N(t)-1} \to \mu \quad (4)$$

However, $S_{N(t)-1} \le t < S_{N(t)}$ for all t. So, it is easy to see

$$\frac{S_{N(t)-1}}{N(t)-1}\frac{N(t)-1}{N(t)} \le \frac{t}{N(t)} \le \frac{S_{N(t)}}{N(t)} \quad (5)$$

Take the limit for t to infinity, then both sides of the inequality approaches to μ, which implies (by Squeeze Theorem) $\frac{N(t)}{t} \to \frac{1}{\mu}$. Plus the definition of renewal function, we have

$$\lim_{t\to\infty}\frac{V(t)}{t} \geq E\left(\lim_{t\to\infty}\frac{N(t)}{t}\right) = \frac{1}{\mu} \tag{6}$$

We also have $V(t) \leq U(t)$, $V(t)$ is a special case of $U(t)$ when $y_0 = 0$, renewal epoch S_n are smaller for the pure case, because non-pure renewal process has a period of time before renewal, thus pure renewal process is renewed more often than the non-pure one for the same amount of time, thus

$$\lim_{t\to\infty}\frac{U(t)}{t} \geq \lim_{t\to\infty}\frac{V(t)}{t} \geq \frac{1}{\mu} \tag{7}$$

Since renewal epochs S_n are smaller for the pure case and hence more likely to be less or equal to t. So we need to show $\lim_{t\to\infty}\frac{U(t)}{t} \leq \frac{1}{\mu}$. It suffices to assume $y_0 = 0$. Fix $a > 0$ and define $y_i^* = min(y_i, a),\ i \leq n, S_n^* = y_1^* + y_2^* + \cdots + y_n^*$. y_i is a sample obtain from Y_i. Define N_t^* to be corresponding pure renewal process with renewal function $U^*(t) = E(N^*(t))$. Since $S^*(n) \leq S(n)$ for all n, $U^*(t) \geq U(t)$ for all t. Before we get the result, we need another substitution equation to represent the $S_{N(t)}$ by y and $N(t)$.

$$E\left(S_{N(t)}\right) = E(Y_0) + E\left(\sum_{n=1}^{\infty} Y_n 1_{N(t)\geq n}\right)$$

$$= E(Y_0) + \sum_{n=1}^{\infty} E(Y_n)P(N(t) \geq n)$$

$$= E(Y_0) + E(Y_1)E\left(N(t)\right) \tag{8}$$

Since we have assumed $y_0 = 0$, we have $E(S_{N(t)}) = E(y_1)\, E(N(t))$. Furthermore, $S_{N^*(t)} \leq t + a$ because $S_{N^*(t)-1} \leq t + a$, $S_{N^*(t)} - S_{N^*(t)-1} \leq a$. Computing

$$\frac{U(t)}{t} \leq \frac{U^*(t)}{t} = \frac{E\left(S_{N^*(t)}\right)}{tE(y_1^*)} \leq \frac{1}{E(y_1^*)}\left(1 + \frac{a}{t}\right) \to \frac{1}{E(y_1^*)} \tag{9}$$

Therefore, we have

$$\lim_{t\to\infty} sup\frac{U(t)}{t} \leq \frac{1}{E\left(min(y_1, a)\right)} \to \frac{1}{\mu} \quad as \quad a \to \infty \tag{10}$$

Here, a can be interpreted as a portion of t. Finally, we can define the renewal equation.

Theorem 6 *(Renewal Equation) Let z(t) be a real valued function, vanishing on (−∞,0) and G(y) is a distribution function, vanishing on (−∞,0). The renewal equation for (z, G) is, for some function Z(t)*

$$Z(t) = z(t) + \int_0^t Z(t-y)G(dy) \tag{11}$$

or in short, Z = z + G ∗ Z. Either or both of z and G could be discontinuous at 0.

Functions related to a renewal process may satisfy such an equation; the objective is to solve for *Z* or at least to approximate it. For example, for any renewal function *V* (*t*), we have

$$V(t) = \sum_{n=0}^{\infty} F_0 * F^{*n}(t) = F_0 + \sum_{n=1}^{\infty} F_0 * F^{*n}(t)$$

$$= F_0 + F * \sum_{n=1}^{\infty} F_0 * F^{*(n-1)}(t) = F_0(t) + F * V(t) \tag{12}$$

here, $z(t) = F^{*0}(t)$ and $G(t) = F(t)$. Also, it is easy to get

$$U(t) = F * 0 + F * U \; where \; F * 0 = 1 \tag{13}$$

What we need is not just to have an equation, we need to solve the distribution.

Theorem 7 *Assume* $G(\infty) < \infty, G(0) < 1$ *and z is locally bounded. There exists a unique solution Z to the renewal equation* $Z = z + G * Z$ *such that Z is locally bounded and vanishes on* $(-\infty, 0)$. *The solution is Z = u ∗ z, where* $u = \sum_{n=0}^{\infty} G^{*n}$.

Assuming U ∗ z is bounded, we check the renewal equation, put Z = U ∗ z to the renewal function

$$Z = z + G * Z = z + G * (U * z) = z + (G * U) * z = (1 + G * U) * z = U * z \tag{14}$$

Specifically, for renewal function with exponential waiting time, for example, if *G* = *exp*(*α*), then 0. So the solution for the corresponding renewal equation is

$$\begin{aligned} u * z &= \int_0^t z(t-x)u(dx) = z(t)U(0) + \alpha \int_0^t z(t-x)dx \\ &= z(t) + \alpha \int_0^t z(x)dx \end{aligned} \tag{15}$$

Theorem 8 *(Key renewal theorem) Suppose $y_0 < \infty$, $\mu = E(y_1)$ and z(t) directly Riemann integralable (dRi). Let $u = \sum_0^\infty F^{*n}$, then*

$$\lim_{t\to\infty} u * z(t) = \frac{1}{\mu}\int_0^\infty z(x)dx \tag{16}$$

3.3 PARKING MODEL PRELIMINARIES

We can model each truck driver's behavior as an alternating renewal process. We assume that all functions and sets that are mentioned are measurable with respect to the appropriate σ $-algebra$ space. Let H_i and R_i each be identical independent distributed (i.i.d) positive sequences, with respective non-lattice distribution F and G, and independent with each other. Here F denotes the distribution of driving time and G represents the distribution of rest time of each truck driver, respectively. For general modeling purposes, the distribution is not specified here.

Let $Y_k = H_k + R_k$, $S_n = \sum_{k=1}^n Y_k = \sum_{k=1}^n (H_k + R_k)$ with corresponding renewal process $N(t)$, which refers to the number of cycles. Y_k is defined as the length of one cycle (driving plus rest). Assume $S_0 = y_0 = 0$ and the renewal is counted at 0. In other word, we treat this process as a pure renewal process, and we do not distinguish short or long rest anymore. An alternating process for the trucking problem can be defined as

$$X(t) = 1_{S_{N(t)}-1 \le t \le S_{N(t)-1} + L_{N(t)}} \tag{17}$$

$X(t)$ is alternatingly "driving" (equal to 1) for times with length H_k and "rest" (equal to 0) during rest periods of length R_k, starting with an "driving" period. In other words, $X(t) = 1$ indicates the truck driver is driving and $X(t) = 0$ indicates the truck driver is rest. We define the remaining legal driving time as $L(t)$. Here Y_1 is a drive-rest cycle and equals to $S_1 = H_1 + R_1$. When we observe trucks on a highway segment, truck driver is driving. H_1 is a random variable, we do not know the value until we observe the truck driver stops for the first time.

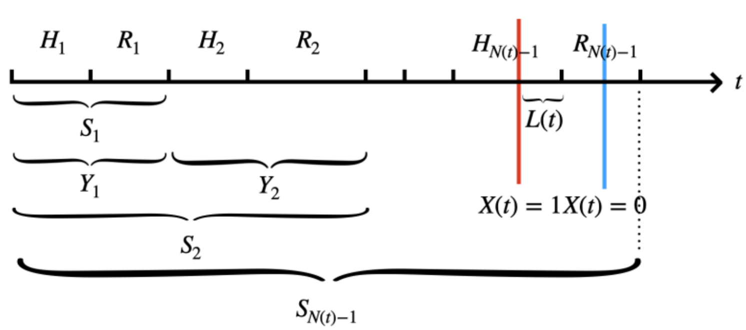

Figure 2: Sample plot for $\boldsymbol{H_t}$, $\boldsymbol{R_t}$, $\boldsymbol{S_{N(t)-1}}$, and $\boldsymbol{X_t}$

Figure 2 shows the details. We can interpret the equation (17) as the following: if a truck driver is in his or her first cycle, in other words, if the truck driver is still in the first stage of the trip, his or her remaining legal driving time is simply $H_1 - t$, where *t* is the time he or she has been driving in the truck. If the truck driver is not in the first cycle, in other words, the time that starting driving is greater than the first cycle y_1, the remaining driving time has a recursive form. We can make this assumption because the expect length of a cycle is the same across the trip. Then, we will develop the cumulative distribution function of $L(t)$.

When the truck driver is driving ($X(t) = 1$) and the probability that remaining driving time is greater than a value *x* is$P(L(t) > x, X(t) = 1)$. Let $U = \sum_{n=0}^{\infty} (F * G)^{*n}$, for any $x \geq 0$,

$$P(L(t) > x, X(t) = 1)$$

$$= P(H_1 - t > x, y_1 > t) + P(L(t - H_1 - R_1) > x, y_1 \leq t)$$

$$= P(H_1 > t + x) + \int_0^t P(L(t - y) > x, X(t - y) = 1)(F * G)(dy) \qquad (18)$$

It is easy to observe the probability formula is consist with the renewal equation form. Solve the renewal equation,

$$P(L(t) > x, X(t) = 1) = U * z = \int_0^t z(t - u)U(du)$$

$$= \int_0^t P(H_1 > t - u + x)U(du) \qquad (19)$$

The probability of observing a trucker is driving is easy to get, just plug $x = 0$ into the above equation:

$$P(X(t) = 1) = P(L(t) > 0, X(t) = 1) = \int_0^t P(H_1 > t - u)U(du) \qquad (20)$$

Take different value for each remaining driving time *x*, we can simulation the distribution of remaining driving time. When $t \to \infty$, we can observe the long-term probability, which is the stationary state after long time of operation. Using the theorem 9 (key renewal theorem) to get the limit probability of the joint probability and the condition probability, respectively:

$$P(L(t) > x, X(t) = 1) \to \frac{1}{E(H_1) + E(R_1)} \int_x^{\infty} \big(1 - F(u)\big) du \qquad (21)$$

Thus there should be a parking facility at the place where the drivers have the lowest probability density of remaining time. In other word, we are able to get the probability $P(L(t) < x, X(t) = 1)$ and derive the cumulative distribution function (CDF) for a truck driver on a highway segment. Then, it is easy to calculate the (limited) expected remaining legal driving time:

$$E(L(t), X(t) = 1) = \int_0^\infty P(L(t) > x, X(t) = 1)dx = \frac{E(H_1^2)}{E(H_1) + E(R_1)} \tag{22}$$

The solution meets our initial idea. After a significant long time, the remaining driving time will enter a statistically stable state. Data can validate the results in the future. To summarize, the idea is to get the overall driving time distribution, then calculate the violation probability and expected remaining driving time. If we are able to acquire the distribution of driving hours and rest hours information of all truck drivers in a certain area, we can estimate the maximum spacing between parking facilities by using the equation. Equation (22) ensure the maximum driving time between each parking facility. For example, $E(H) = 6$ with $Var(H) = 1, E(R) = 12,$ then $E(L(t), X(t) = 1) = 3.1\ hr$, which indicates the maximum interval of travel time in the parking lot is 3.1 hr. However, such data may be difficult to acquire accurately. We will introduce a simplified model based on the key renewal theorem in the next section.

4.0 PARKING DENSITY MODEL

4.1 BASIC MODEL

Instead of focusing on the lump sum capacity of discrete rest areas along the highway, the perspective to start here with is on the needed density of parking space along a highway for an (randomly chosen) individual trucker. The driving time is comprised of cycles of H_i and R_i, where H_i represents the driving hours and R_i the rest time in the i^{th} cycle, both being random. Each cycle has a length of $H_i + R_i$, H and R alternate with each other. Here H_i is *i.i.d.* for all i. We use H for the random variable of a common probability distribution over all the cycles i. In the same way, we use R for the random variable of a common probability distribution over all the cycles. Therefore, one may treat H and R each recurrent in a renewal process. The driving distance between two consecutive rests, assuming a constant driving speed v, is therefore *i.i.d.* distributed, and is equal to vH_i. Additionally, let $h = E(H_i) = E(H)$ denote the mean driving hours each cycle, and $r = E(R_i) = E(R)$ the mean rest hours, for any driver i.

Proposition 1

If the traffic volume is denoted by V, the total parking hours needed have a density requirement along the highway, in an ideal *situation, expressed as* $\frac{Vr}{24vh}$.

The result is straight forward from the renewal theory. $\frac{1}{\mathrm{vh}}$ is the density of parking capacity in terms of stops per unit of highway distance for a single driver, while $\frac{\mathrm{r}}{\mathrm{vh}}$ represents the density of stall hours for a driver. Multiplied by the volume and divided by 24 hours per day, $\frac{\mathrm{V}}{24}$, it represents the stall density of rest areas in terms of number of stalls per unit length of distance averaged over the roadway for all drivers. Note that each stall may serve trucks for 24 hours in a day. This proof assumes a uniform distribution of rest hours during a day and ignores the heterogeneity between different hours of the day at rest areas, which needs to be addressed separately later.

There are a few abnormalities to consider in order to propose a meaningful formula.

- Traffic peaking effect, which is similar to the peak hour factor for the urban commuter traffic. The parking at particular spots may need to be larger than the expected value by a factor of β.
- Level of service factor. The expected rest hours used for the calculation only on the average approximately satisfies 50% of the demand leaving the rest 50% unsatisfied, which is not acceptable. The capacity provided shall satisfy a demand at such a confidence as 95%.

Therefore, a confidence factor z_α shall be incorporated, where α corresponds to a conference of $1 - 2\alpha$.

As a result, the parking density provision along the highway shall be estimated as follows.

$$\frac{z_\alpha V r}{24\beta v h} \quad (23)$$

The peak hour factor β may be calculated as the ratio between the daily average density over the peak parking hour density needs, it ranges between 0 and 1.0. Note that this peak hour factor is regarding the peaking of parking needs instead of peaking of truck volumes.

4.2 SPECIAL SITUATIONS

4.2.1 Lump sum provision of parking capacity

Trucking parking capacity is not provided in a continuous way along the roadway, but in lump sum with a spacing as in the reality.

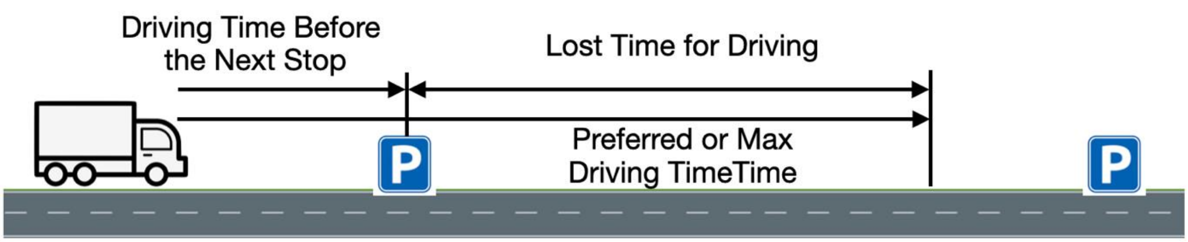

Figure 3: Lost Time due to Parking Area Spacing

Due to the lump sum provision of parking, truckers are not able to drive to their very limit of hours either mandated or preferred before they come to a rest. As it is shown in **Figure 3**, they need to decide whether their remaining driving hours will allow them to reach the next rest area if they miss the current one. They will come to a rest area if they know they cannot reach the next area, forfeiting their effective remaining driving hours. The tradeoff is that if the public sector decides to construct parking areas with larger spacing along the roadway, they will need to provide a larger capacity because truckers, considered all being law abiding citizens, would lose more of their effective, legal driving time for having to rest earlier before the end of the remaining driving hours. We talk about road end effect and the network effect respectively in the following.

4.2.2 Road end effect

There are two factors to consider regarding road end effects. One is average hours lost and the other is unequal probability distribution of remaining hours at locations such as suburbs of major metropolitan areas. Earlier we assume that the remaining hours distribution of truckers remains identical over the locations of roadway. We talk about these two effects in more details below.

4.2.3 Average hours lost

Suppose a driver's longest preferred and possible driving distance would end between two parking areas with a spacing of δ. Because the origin of this driver's this cycle of driving may be anywhere with equal probability, which means there is no recorded or proven knowledge about which place being more likely than others to have originated the trip. This is especially true when the traffic is on a network and the trajectories are intertwined. In this case, this driver's longest preferred and possible travel end may be considered uniformly distributed between the two consecutive stations. Note that this travel end is a stop for rest or end of the shipment.

Between two rest areas A and B longitudinally positioned along a highway, a trucker stops for rest at location A when the trucker's remaining driving hours is not enough to drive to location B. If allowed, the trucker may stop at any location between A and B. The would-to-stop locations of the trucker is an interesting subject of investigation here. Because the starting point of the driver's driving is not known, we assume drivers passing a spot x_1 has a remaining hour distribution be identical to the remaining hour distribution of those drivers passing the spot $x_1 + \delta$, where $\delta > 0$, which means that drivers passing any spot along the roadway have identical remaining hour's distributions. This means the probability of a driver that has to stop at a spot within two rest areas for rest remains constant over the roadway. In other words, any spot between A and B has an equal probability of being a would-to-stop location. This is valid when no knowledge about upstream rest area distributions, particularly true in the (especially dense) network situation. In the case of a single roadway line with known upstream rest areas in sequence, a similar result may still hold. Therefore, the result in Equation (23) becomes the following

$$\frac{z_\alpha Vr}{24\beta(vh - 0.5s)} \tag{24}$$

Equation (24) is the density of parking provision longitudinally along the roads in terms of the number of stalls per mile of roadway, where *s* is the average rest area spacing along the roadway. Note that in the general case of interwoven network of roadways where traffic randomly entering and exiting the roadway system, it makes sense to assuming an averaged lost driving distance due to discrete availability of parking areas is *0.5s*. However, it remains arguable in the case of a single line of roadways. Nevertheless, one may use *0.5s* as an approximate to maintain the problem tractable. Better means may be further explored along this line of research.

4.2.4 Unequal probability distributions

One special situation is due to the fact that highways do not have an infinite length, which gives rise to the end effect or boundary effect. This end effect means that traffic leaving a certain area such as a metropolitan area like Chicago or Houston do not have the same probability distribution of remaining driving hours as in other areas such as middle of two metropolitan areas or areas nearing their destinations. A line of highway with unevenly distributed traffic may also give rise to a similar situation.

How to deal with the end effect? The answer is that as long as the probability distribution of the driving hours is known, which may be in one form for areas close to the departure from a metropolitan area and in another form in areas far away from the business areas, one may estimate the average, respective area specific parking provision density needed in order to maintain a certain service level in terms of satisfying parking demand.

4.2.5 Network effect

Consider that truckers do not drive on a single roadway of infinite length but on a network with a certain density, although roadways may still be some significant distance apart from each other. Assume that the truckers drive on it with random entry/exit locations and the entry/exit locations are equally likely distributed on the network. This somehow supports the initial assumption of the study about infinite length of highways because traffic 'loop' around on the network. There are a few cases to discuss for this situation as follows.

- **Uniformly distributed spatial traffic**
  In this case, each road has the same probabilistic volume and the OD locations are uniformly distributed in the geographic area. The above Equation (24) applies.

- **Spatially uneven traffic**
  In this case, traffic or their OD locations are not evenly distributed spatially. Network needs to be divided into subnetworks of respective uniformity of traffic. Equation (24) applies to each subnetwork then with its own parameters accordingly.

With spatially unevenly distributed traffic, is there a simple way to plan for the parking provision? This is discussed in the next subsection. Let first consider the first case with uniformly distributed traffic on the network, uniform regarding the spatial distribution of OD locations and travel time. Consider an area with a dense network of highways on which large volume of trucks travel. Assume that we are dealing with an area, a measure of whose traffic is vehicle miles traveled (VMT), which is a report measure from local, and state to the federal governments. In dealing with a region or area in which there is a roadway network, one may readily reach the following equation.

$$Parking\ density\ = \frac{r z_{\alpha} M}{24 \beta d \Gamma} \tag{25}$$

Whereas M represents the vehicle miles traveled (VMT) on the network of interest. Γ is the total roadway (not lane) miles of the network; h is the average driving hours of vehicles between two rests in which the vehicle stops for a rest, here $d = vh - 0.5\delta$. Rest area spacing and size are a trade-off in practice. Current spacing of rest areas is about 1-2 hours of driving on the interstate highways. Regarding inputs to the Equation (24), traffic volume may not be readily available for every highway segment, VMT is sometimes the only available performance of truck traffic in a given (even local) area. Therefore, Equation (25) may find its convenient application in this situation. Equation (25) allows to divide a large area into smaller zones, for each of which a parking density may be calculated, which may represent a practical means to deal with heterogeneous traffic distribution on the network.

There are two cases regarding using Equation (25) as follows.

- $\frac{\delta}{vh} \approx 0$. In this case, Equation (25) applies with little error.
- $\frac{\delta}{vh} \gg 0$. In this case, the spacing parameter δ used in Equation (25) shall be consistent to the resulting spacing parameter implied in the result from Equation (24).

In a general sense considering an idealized situation where the truck parking areas are of identical sizes, one may multiply both size of Equation (25) with δ to get to the following result:

$$Parking\ size = \frac{r z_\alpha M \delta}{24 \beta d \Gamma} \quad (26)$$

In Equation (26), the left side means the number of parking stalls. Once a parking area spacing distance is determined, the parking area capacity may be determined accordingly.

4.3 LOCATION DECISION TRACING TRAFFIC DENSITY

Another lesson would be to locate the parking areas near interchanges between major roads, where the *local* area (around the interchanges) density of VMT is higher than only considering location distribution along single roads in a separate manner. This implies two means of parking location planning. The first is positioning rest areas by tracing the network traffic density, while the second is to decide location by only considering individual roadway lines longitudinally. Even if the longitudinal density of truck parking needs remains similar for each section of the road, the *local* area density of parking needs may be different between the two location decisions above. A simple proof is obvious if one calculates the enclosed VMT and then the parking needs densities. Each pair of rest areas in the middle of a road between interchanges in both directions is considered one location, as indicated by Plan 1 in **Figure 4**. Each pair of locations at the diagonal corners of an interchange is also considered one location, as indicated by Plan 2 in **Figure 4**. Suppose roadway is a n by m grid network with road separation at a constant distancing s.

- Case 1: all rest areas are located in the middle of the road sections between road intersections, and each road section has one and only one parking area.
- Case 2: parking locations are all at the intersections, and each intersection has one and only one location.

Again, each rest location contains two spots at opposite sides of the road or at diagonal corners of the intersection. In case 1, the total number of parking areas is *2mn+m+n* while case 2 has a total parking area equal to *mn*. Both case 1 and 2 ensure an exact driving distance of *s* between parking areas for any driver along any route on the network. An advantage of Case 2 is that the size of each parking area at the interchanges almost doubles that in case 1. If one checks on the traffic density around each parking area defined by a square centered at the parking area with a side size *s*, one may find that case 2 locates the parking areas with high traffic and is equivalent to pooling servers together. Its performance in terms of serving the parking needs shall be higher than case 1 because of an underlying mechanism that two separate queuing systems have a poorer performance than the two pooled together to have one single queuing system.

The basic concept of the above discussion is that there is an overall parking needs for an area with a certain VMT. Locales of the parking areas within the area shall trace the *local* VMT. *locals* with higher VMT densities shall be accommodated with a higher parking density than other locations where VMT is lower. In this case, the overall service of the parking areas would be better.

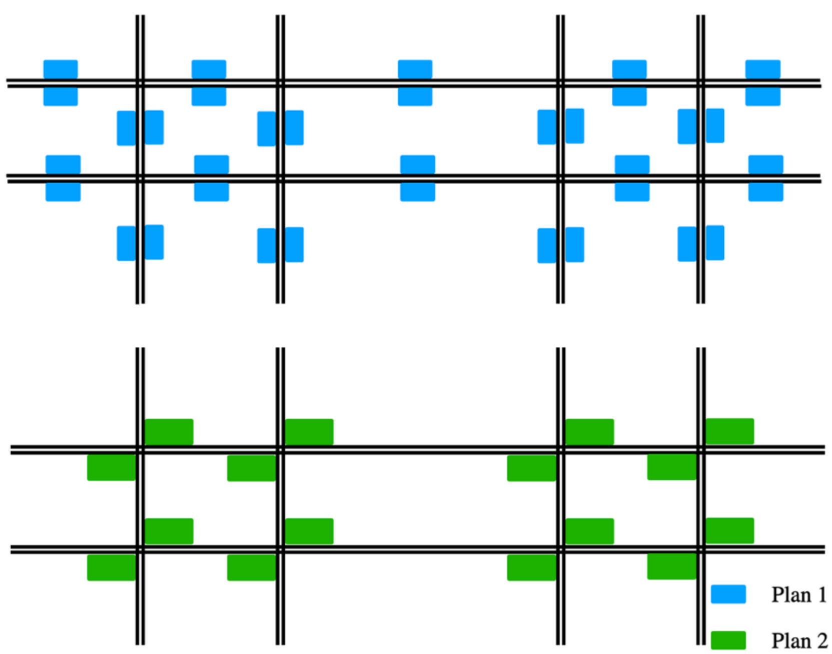

Figure 4: Network rest area location examples: tracing VMT for better service

4.4 PARKING SPILL AS AN INDICATOR OF LEVEL OF SERVICE

When a trucker gets into a parking area on the roadside to rest by the HOS rule and finds no stall available, the trucker is forced either to continue driving in violation of the hours or to park nearby illegally. We term this *spill*.

The driving and parking times are both assumed random. So is the traffic volume. Therefore, the number of trucks needing to park at a location at any time of the day is random, so is the number of stalls available to incoming truckers at a given parking area. The spill is considered and addressed in setting the parameter α and z_α in the earlier equations. The actually observed spills may be used to calibrate the α values used in the equations. Noteworthy is that theoretically it is impossible to completely eliminate spill. The only reasonable way to approach it is to control the probability of spill to be under a certain threshold such as 5%. A practical challenge remains as to how to measure the probability of spill. One way is to observe the number of trucks that have entered the rest area but without a parking stall available so that the truck either has parking on the ramp or has left the rest area. In other words, it is important to track the total number of entering trucks minus the exiting ones during a period as compared to the total number parking stalls available. The difference between the entry and exit traffic continuously tracked represents the trucks parked inside the rest area. Currently, few parking areas in the U.S. have such capability or have conducted such a practice in our knowledge. This has made it almost impossible to verify our models proposed here.

5.0 EXAMPLES

5.1 FLORIDA EXAMPLE

Here we use data from the Florida Department of Transportation to illustrate the use and implications of our proposed models.

Based on the average truck AADT of full roadway segments of Florida (data source: https://www.fdot.gov/statistics/gis/default.shtm\#Traffic), $V = 1137$, $\beta = 1$, $v = 65$ mph, $r = 5$ *hr*, $h = 5$ *hr*. The average number of hours of driving per driver is difficult to obtain due to the potential privacy implications, so we subjectively estimate the average to be 5 hours, likely slightly higher than the true value. The rest time length is estimated to be 5 hours over the day and night time based on information in **Figure 5** Assuming $z_\alpha = 1.645$ (95 % confidence level):

$$\text{Parking Density} = \frac{z_\alpha V r}{24 \beta v h} = \frac{1.645 \times 1137 \times 5}{24 \times 1 \times 65 \times 5} = 1.20 \tag{27}$$

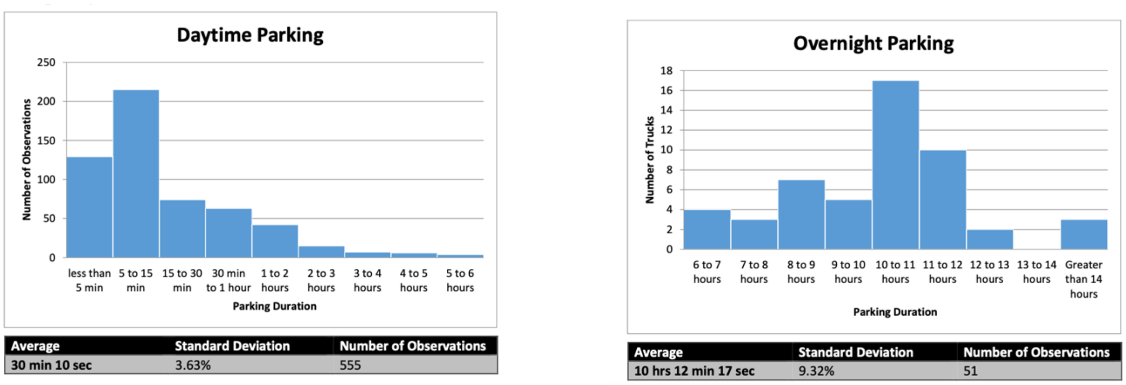

Figure 5: Truck parking hours distributions (From 8/18/16 to 8/20/16 and from 9/4/16 to 9/8/16)

According to report (Office of Freight Management and Operations, 2015), the existing parking space density is 1.043 in Florida, which indicates an approximate 13% shortage. Note that the shortage comes out of calculation that ignores the private parking. The actual private parking may be sufficient to overcome the calculated shortage calculated by only using public rest area data. Also remember that this density is according to a 95% satisfaction of a parking need when it arises, which corresponds to an occupancy of the parking stalls at about 61%, not a high value. With errors in the parameters, this result might indicate a sufficient parking capacity from the public facilities in the study area of Florida, although the supply of private parking needs yet to be examined.

5.2 CHALLENGES

The daytime need hours may be different from the nighttime. For daytime, the average driving hours probably is around 5, or may be slightly longer. The rest duration during daytime is much shorter. Equation (25) may be deemed suitable for daytime needs.

What about the nighttime needs? At nighttime, the traffic volume is much lower than in daytime. For nighttime, what traffic AADT should be used? Kansas DOT and KTA (2016) found that truck rest area peaking happened during 12AM to 4AM for overnight parking. This finding makes sense because this peaking period is one that the truckers rest time overlap the most, meaning truckers likely entered the parking areas from 7-9pm all the way to midnight. Early truckers left the rest area as early as 4-5AM.

Night parking is needed for long distance truckers. For truckers who drive within 8-10 hours for the business two-way would not likely need to park overnight on the road. Therefore, calculation of the overnight parking capacity would only need to consider the volume of long-distance truckers. In contrast, the daytime parking needs would need to consider all the long-distance truckers and short distance truckers beyond 4-5 hours of driving business. Drivers who finish the delivery business within 4-5 hours generally would not need a rest normally. In addition, local truckers, who start out at the beginning of the day and get home at the end, even if they drive more than 8 hours on a day, would not need to park overnight because their one-way trip would be within 4-5 hours. To conclude, it'd be meaningful to differentiate the long-distance truckers from the local/regional drivers in order to decide the overnight parking space need.

If we treat long haul traffic with the same formula as Equation (25), it'd reveal the density needs for (largely) overnight parking during the period from 7pm to 7am. In this case, the remaining hours of driving for each driver might be much less than assessed at a moment during the daytime. Each driving, after the overnight parking, will have a remaining hour of driving much larger than before parking.

6.0 CONCLUSION

Due to the hours of operations regulation, truckers in the U.S. as well as in the EU countries are required to rest periodically for the sake of safe driving. Rest areas must be available to truckers when needed. However, the tremendous challenge arises from here due to the randomness in the origin-destination of each trip, demand for shipping, individual preference to the rest, and driving hours, as well as different subjective considerations of local traffic in the planning of their driving hours. Therefore, it remains a significant challenge as to where to locate rest areas and in what capacity.

Most literature on truck parking issues focuses on the diagnosis of locations with truck parking issues as indicated by roadside parking, ramp parking, and other illegal parking behaviors. Little analysis is conducted to identify factors affecting trucking parking needs and the relationship between these factors and the truck parking demand. Although it is a great challenge to study this analytical relationship, this paper represents the first step in this direction. We believe the effort to exploring this relationship is in the right direction. Along this direction of effort, we propose analytical formulas in this paper for the relationship between identified factors and the parking demand as indicated by demand density along the roadway. This relationship considers traffic volume as represented by VMT, rest and driving hours distributions, traffic speed, rest areas level of service indicated by the probability that a trucker is able to find an available stall in the parking lot when a need for parking arises, etc.

This study marks the beginning of a line of effort to address parking capacity planning. Much more remains to be studied. For example, the probability distributions of rest and driving hours need to be calibrated using field data. Due to the time and resource limit, we do not conduct a full-fledged study of the time distributions in this study, and we leave it as a future effort. As a result, it is hard to test our model except for applying it to explaining in a general sense the adequacy of parking capacity in an area. In this study, we used the Florida state department of transportation data and revealed a general shortage of parking space along the Florida highway; However, our initial analysis using the Florida data does not consider supplementary parking spaces offered at private parking areas, among the others. One side result, also very interesting, is that a 95% level of service to truckers, meaning they are able to find a spot available at 95% chance when it comes to a parking area for rest, implies a 60% occupancy of parking stalls on average, which sounds like a waste of public resources. In other words, in public planning of truck parking areas, high occupancy shall not be in consideration because it implies a high probability of parking spills.

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